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ABSTRACT

Two concerns labeled as fundamental to the field of curriculum are identified: (1) the identification of educational content, and (2) the development of ways to make it available to young people. These factors are reviewed from the perspective of their influence on the mathematics curriculum, with note that the critical integral relationship between them is inadequately addressed by curriculum theorists. An eclectic approach to find the differences and commonalities among the disciplinary theories of knowledge is advocated. Besides suggesting that curriculum theorists reconsider the two fundamental concerns in relation to the mathematics curriculum, mathematics should illustrate points that apply to the other areas of subject matter. Integrating the selection of content and designing a presentation of that content is viewed as subject-matter or concept specific, while each factor separately can be applied more directly to a variety of disciplines. (MP)

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Conceptual Change Analysis:
Implications for Mathematics and Curriculum Inquiry

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Conceptual Change Analysis: Implications for Mathematics and Curriculum Inquiry

Huebner (1976) suggested that the field of curriculum is suffering from too much diversification. He wrote:

> Since the 1960's, the curriculum family has included those interested in content, method, teacher education, human development, and freedom, progressivism or conservatism, educational technology, evaluation and educational objectires or purposes. (p. 156)

The solution which he advocated was a decrease in diversification by a return to two fundamental concerns:

1. the identification of educational content.
2. the development of ways to make it available to young people.

In suggesting a reemphasis of these two concerns, Huebner has not relieved us of the diversification problem, for it is undoubtably the pursuit of these two concerns which lead to where we are now. However, what his reemphasis can do is to suggest considering these two concerns as starting points in curriculum inquiry, and systematically expanding on them to include other concerns.

What I will do in this paper is to begin with an examination of these two concerns, and suggest that their definition needs to be reconsidered. In addition, I will add a third consideration which I think has been inadequately addressed, and often, in fact, ignored by curriculum theorists; that is the integral relationship between identifying educational content and deciding how to make it

available to young people. It is the particulars of this relationship which I think are subject-matter specific, if not to a large extent, concept specific, and hence must be undertaken with respect to one's subject-matter. To illustrate my points, I will refer specifically to mathematics education, but I will also outline how the argument applies to other subject-matters.

The Identification of Content

In the phrase, "the identification of content", is buried an assumption about the nature of knowledge which is fundamentally unsound, and which, if accepted, can lead to the fragmentation of the curriculum field as described by Huebner. This assumption is that knowledge exists as an immutable, objective and eternal body existing as independent from people. If this assumption is accepted, then the task of the curriculum specialists is to select the appropriate knowledge from that body and it is only in this selection that questions of values, purposes and authority in education gets injected. I suspect that the omission of a discussion of questions of value, purposes and authority from the basic examination of knowledge leads us to speak as though ideally "the identification of content" is undertaken through only a consideration of the aims of education and the learners to be taught. If this were so, curriculum could ignore the particularities and integrity of the various disciplines. I will argue that such an objective and external view of knowledge is both naive and inaccurate. Furthermore, I think that this erroneous basic assumption and the accompanying failure to examine critically the

various theories of knowledge inherent in each discipline have obfuscated the task of curriculum theorists and rendered much of their work inconsequential.

The importance of careful examination of one's implicit theory of knowledge is being called for from various corners, although in education, it is most likely to be heard from the subject-matter specialists, who have mastered and examined their own disciplines and have found the same basic misconceptions (Schwab (1960), Cherry-holmes (1980). Philosophers of education are also calling for such an examination, (Peters (1977), Scheffler (1968), Strike (1976), Soltis (1979)). Soltis (1979) wrote:

> The point, again, is to see that the more adequate our grasp of what we understand as 'knowledge', the more we can consciously, responsibly, and morally play the role of the educator." (p.3)

Others have emphasized the importance of examining one's theory of knowledge in particular disciplines such as science, Kuhn (1962), Popper (1972), Toulmin (1972), and Elkana (1977); in mathematics, Lakatos (1976), Hersh (1979); in literary criticism, Booth (1979). The ability to reflect on the theories of knowledge is a unique characteristic of people, as Toulmin (1972), succinctly summed it up writing, "Man knows and he is also conscious that he knows". (p.1). It is this reflection on our ability to know and what it is that we deem to consider knowledge which ought to constitute the core of the curricular inquiry.

In the philosophy of science, one can distinguish the evolution of three theories of knowledge which I will refer to as: absolutism, progressive absolutism and conceptual change.

Absolutism portrays knowledge as commulative and has its roots in empiricism and positivism. According to such a theory of knowledge, knowledge is comprised of the objective accumulation of facts. Knowledge is value-free and theory-free. New knowledge is simply added to the store of previous knowledge. The 'absolutist' view of knowledge is connected closely with verificationism in which the test of truth is restricted to observation. Theoretical constructs are excluded, because no objective value-free method of determining their truth or falsity exists.

The second theory of knowledge, which I am calling 'progressive absolutism', was first presented by Sir Karl Popper. According to Popper (1972), knowledge progresses by the successive replacement of theories by more powerful and all-encompassing theories. A new theory accounts for all the data that the previous theory accounts for, but further extends it to include data which could not be explained by the previous theory. The same data are available and relevant to both theories, and so "critical experiments" can be constructed to objectively test for the superior theory. He wrote, "the new theory, although it has to explain what the old theory explains, corrects the old theory, so that it actually contradicts the old theory; it contains the old theory, but only as an approximation." (p. 16)

Underlying such a view of knowledge is a basic commitment to an absolute truth, towards which we are striving and forever approaching more closely. There is also the assumption that two theories can be compared side by side and that one can objectively determine the superior theory by its increased potential for explanation.

An examination of key historical transitions between theories fails to reveal these "critical experiments" which objectively determine the superiority of one theory over another. This failure prompted the development of a third theory of knowledge herein referred to as 'conceptual change.' It acknowledges that theories do replace other theories, but suggests that no objective critical experiment exists to determine the superiority of one theory over the other, because these competing theories each rely on different data and interpretations of the data, and hence are "incommensurable." Kuhn (1962) was the first to popularize a conceptual change view of science wherein he proposes a revolutionary account of the scientific development. He theorized that paradigms (conceptual systems) are replaced by wholly new paradigms. Other proposals for accounting for conceptual change were proposed by Toulmin (1972) and Lakatos (1970). Toulmin saw the transition between paradigms as an evolutionary one, whose intensity ranges from the discovery of a new piece of data requiring little or no reconceptualization to transition in which fundamental intellectual ideals were reevaluated. Lakatos suggested that in conceptual systems, there exists a theoretical hardcore surrounded by a protective belt and that the scientist sacrifices theories from the protective belt in order to protect the theoretical hardcore. He described these conceptual systems as "research programmes", and he called them progressive if they are gaining in explanatory potential; if not, they are referred to as degenerating research programmes. Others, such as Elkana (1977) have further explained theories of conceptual change. Three basic tenets tie the conceptual theorists:

1) Knowledge changes and develops; it is not static;

2) The knowledge is not externally defined, but it progresses through a community of scholars who influence its values, its truth conditions and standards of evidence;

3) Theories influence progress and are not objectively comparable in that they strive to explain different phenomena, involve different evidence and interpret that evidence differently.

The importance of these theories of knowledge to the first task proposed by Huebner which was the determination of appropriate educational content can now be explained. If one adopts the absolutist theory of knowledge, then the curriculum theorist is free to select the content at will, depending on his/her purpose of education. However, if one adopts the conceptual change approach to knowledge, then the decision to select any particular piece of knowledge for content carries with it an implicit decision to support a particular system of conceptual beliefs, or research programme. The point is that the determination of content is not a value-free endeavor, and this is not only because education inherently involves values, but also because knowledge too must be redefined to include fundamental epistemological commitments, which are not value-free. It should also be emphasized that the acceptance of a conceptual change approach to knowledge not only injects values into epistemological questions, but it recognizes that progress in a discipline is through its community of scholars, and hence, sociological, psychological and political influences are inherent in the body of knowledge. It exists through its scholars, not independent of them.

At first glance, this seems to leave us in exactly the position which Huebner sought to extract us from, however, I do not believe that this is to be the case. For we have made some progress: 1) we

have extended the location of values and authority questions to the realm of knowledge, and not just in the aims of education; 2) we have recognized the possibility of different conceptions of knowledge generally, and of any particular piece of knowledge; 3) we have undertaken reflection on the competing theories which allows us to avoid 'relativism.' The next step is to see what is the task of the curriculum specialist who have progressed to this point. He/she is still faced with the decision of what is the appropriate content to include in educating students, as well as considering how to make that content available to students.

The determination of these decisions is where I strongly advocate for a subject-matter-focused curriculum theory. Given the various theories of knowledge, one must explore and command his/her own discipline so that the decision of what to teach is based on a thorough understanding of that discipline. This is not to advocate leaving such deliberation to the disciplinary scholar or practitioner, for these individuals are also likely to hold unexamined assumptions about the knowledge of the discipline. One must determine for oneself an epistemology of one's discipline. To make this point more clearly, let me take mathematics and describe it according to the three views of knowledge. In turn, I will describe how these theories of knowledge influence the decision of what content to include.

Most people conceive of mathematics as absolutist. Historically, mathematics was heralded as the epitome of certainity, immutable truths and irrefutable methods. Its claims were secure through

the infallibility of its supreme method, deduction. It is common to see mathematics portrayed as eternal, unassailable and exact. According to such a view, mathematics has no theories which guide, obscure, or determine the course of its development. Its progress is straightforward and clearly defined and its results are assured. Concepts in mathematics do not develop; they are discovered, and the impression is given that their structure is unchanging. An absolutist view of mathematics portrays it as linear or hierarchial, as an accumulation of truths, predominantly free of theoretical assumptions. Two criteria must be met to classify mathematics as an accumulation or absolutist theory of knowledge: 1) the independence of the truth of a discovery from any overlying theory; and 2) the previous truths are left unchanged by the discovery of a new truth. Both of these are met by viewing mathematics as proceeding by an accumulation of mathematical truths and as having an inflexible, a priori, structure. Lakatos (1976), described and criticized such a view of mathematics when he wrote:

> "In deductivist style, all propositions are true and all inferences valid. Mathematics is presented as an ever-increasing set of eternal immutable truths. Counterexamples, refutations, criticism cannot possibly enter. An authoritarian air is secured for the subject by beginning with the fully-fledged theorem and by suppressing the primitive conjecture, the refutations, and the criticism of the proof. Deductivist style hides the struggle, hides the adventure. The whole story vanishes, the successive tentative formulations of the theorem in the course of the proofprocedure are doomed to oblivion, while the end result is exalted into sacred infallibility." (p. 142)

Lakatos continued on to conclude that such a view of mathematics leaves one with the impression that "the mathematical conveyor-belt of truth cannot fail." (p. 143)

An examination of the history of mathematics indicates that the absolutist theory of knowledge in mathematics cannot account for even the most basic reconceptualization. For example, in an absolutist's account, the concept of number as it evolved would be a simple extension from the natural numbers, to the integers, the rational numbers, the irrational numbers forming the real numbers with each extension being simply an addition to the previous number concept requiring no reconceptualization of the previous concept. The Greeks number concept was based on the ratio between geometric magnitures, i.e., the ratio between two-line segments, two planar areas, two volumes. (See Spengler (1925), Toeplitz (1963)) Given such a number concept, it is not surprising that the concept of the number zero, infinity or the irrationals, would be excluded for these cannot be described by a ratio of two integers. The discovery that the square root of two could not be expressed as the ratio of two integers and yet was constructed as the diagonal of a square of side one, resulted in a crisis in Greek mathematics. The Greeks resolved the crisis by excluding irrationals from the domain of numbers and only accepting their existence as a magnitude. However, later, the incorporation of the irrational into the domain of number required not simply an extension of the number concept, but a radical modification of the previous concept of number. Irrational numbers are commonly defined as infinite sequences of rational numbers, which is a far cry from the concept of number as the ratio of two magnitudes, as expressed by the Greeks. Such a reconceptualization cannot be accounted for in an absolutist theory of knowledge for mathematics.

If an absolutist theory of knowledge is not appropriate for mathematics, then perhaps a progressive absolutist theory will suffice. The progressive absolutist asserts that theory influences findings in a discipline, but also, that progress is a process of replacement of previous theories by superior theories which account for all the previous data and more. Each progressive theory approximates truth more and more precisely. Mathematics according to such a view is influenced by its theories, however, progress consists of discovering mathematical truths which are not consistent with a theory or not accounted for in the theory, and then extending the theory to account for this larger realm of mathematical phenomenon. The superiority of the more encompassing theory would be extablished by objective criterion; its ability to explain more mathematics. Theories, according to progressive absolutists of knowledge, can be lined up and compared objectively. The discovery of non-Euclidean geometry illustrates an inadequacy of this theory. In non-Euclidean geometry, Euclid's fifth postulate is replaced by another postulate, resulting in another geometry. Mathematicians struggled to determine the mathematically superior system, only to conclude that the systems are mathematically equally correct by all internal mathematical standards. One system may be selected over the other system for its ability to match extra-mathematical standards, as Einstein selected non-Euclidian, but neither can be argued as more or less true.

The competition between different perspectives of mathematics continues, and the difficulties to devising critical experiments to determine superior theories in mathematics are not limited to the

example given in non-Euclidian geometry. Intuitionists and constructivists consistently argue for different overriding theories for mathematics than do formalists and set theorists. The paradoxes which resurface continually in the foundations are indicators of the same confusion of which theories are more correct. Mathematics like other disciplines has no clear and objective standards for truth which are irrefutable and eternal. Toulmin (1972) wrote:

> ...the development of mathematical disciplines exposes their concepts and methods to transformations as profound in their own way as the natural sciences. Such fundamental concepts as 'validity' and 'rigour', 'elegance' and 'proof', and 'mathematical necessity' undergo the same sea-changes as their scientific counterparts 'soundness', 'cogency', and 'simplicity' 'relevance' and 'physical necessity'. Even the basic standards of 'mathematical proof' have themselves been reappraised more than once since Euclid's time. The result is that the concepts, methods, and intellectual ideals of mathematics are not more exempt from 'the ravages of time'--as Descartes and Frege hoped and supposed --than those of any other intellectual discipline. (p. 252)

If Toulmin is correct, and if no objective and external standard exists to determine the superiority of mathematical theories over each other, then mathematics becomes a candidate for the application of a conceptual change theory of knowledge. The view of mathematics which results from a conceptual change portrays mathematics as having competing theories which are the result of attempts to solve outstanding problems. Toulmin proposed that the history of a discipline be characterized by a genealogy of problems rather than by fixed methods or content. The appropriate methods or content to solve these problems develop in response to the problems, combined with a consideration of the intellectual ideals for the discipline. The image of the discipline which emerges from Toulmin is reminiscent of Wittgenstein's rope analogy, where no single strand extends the entire length of the rope, nor is

there any point where all the strands end simultaneously, but the continuity and diversity of the discipline is still achieved. This is what I conceive of a conceptual change theory of knowledge for mathematics to be like. The attempts to solve certain problems in mathematics motivate the theories, although the theories cannot be derived from the problems in any deductive sense. Progress is the result of conjectures and refutations. Lakatos (1976), suggested this when he wrote:

> Informal quasi-empirical mathematics does not grow through a monotonous increase of indubitably established theorems, but through the incessant improvement of guesses, by speculation and criticism, by the logic of proofs and refutations. (p. 5)

This view of mathematics meets the three criterion for a conceptual change theory of knowledge: knowledge changes, it is theory-laden, and the theories are not objectively comparable.

In summary, the conceptual change theory of knowledge can be applied to mathematics as well as to science. The view of mathematics which emerges is that mathematics is more than an accumulation of the truths, but that it involves a development of ideas which is not theory-free and hence, not value-free. Its truth is not external to its scholars, but is a result of the interactions between competing theories, expectations, intellectual ideals and previous knowledge. As Hersh (1979), so bravely put it:

> It is reasonable to propose a new task for mathematical philosophy; not to seek indubitable truth, but to give an account of mathematical knowledge as it really is--fallible, corrigible, tentative and evolving, as is every other kind of human knowledge. (p. 43)

Implications for Curriculum Theorist

If the curriculum theorist accepts a conceptual change theory of knowledge in mathematics, then the question is what are the implications for the first task in curriculum design: the determination of content. This can be answered at two levels:

1. The basic tenet is that what one teaches ought to reflect the theory of knowledge which one thinks it most appropriate and adequate for that discipline. This means that one ought to select content which accurately portrays the particular discipline involved. For mathematics, it means that mathematics ought to be portrayed evolving, growing, and changing, not as static immutable truths.
2. An analysis of particular concepts in the discipline yields a variety of ways of conceiving of a particular concept, as well as a variety of ways in which those concepts develop. Therefore, the curriculum theorist must also consider alternative conceptions of particular concepts to assess their appropriateness for inclusion as content.

The first implication of accepting a conceptual change theory is that the selection of content requires one to accurately portray ones discipline. The assumption here is not that there exists a singular accurate portrayal of a discipline, but that if one thinks that controversial conceptions of a discipline exist, then the controversy is also appropriate content. In mathematics, a conceptual change theory commits one to exploring the development of the concepts, and seeing how they have evolved over time. It is difficult to advocate

for such an approach to mathematics, because historical examination of mathematical concepts has also lacked this developmental perspective. Most history of mathematics is a cataloguing of discoveries, as if a precise determination can be made. It is often insistent on using modern mathematics as an appropriate standard to judge past mathematics. For a criticism of the conventional history of mathematics see Unguru (1975).

The importance of the reexamination of the history of mathematics is that the mathematics educator has been caught in the untenable position of criticising formalism in mathematics education solely on pedagogical grounds. As a theory of knowledge, formalism for mathematics is inadequate mathematically as well as pedagogically. Hersh (1979) wrote:

> The criticism of formalism in the high school has been primarily on pedagogic grounds: 'This is the wrong thing to teach, or the wrong way to teach.' But all such arguments are inconclusive if they leave unquestioned the dogma that real mathematics <u>is</u> precisely formal derivations from stated axioms. If this philosophical dogma goes unchallenged, the critic of formalism in the schools appears to be advocating a compromise in quality: he is sort of a pedagogic opportunist, who wants to offer the student less than the "real thing." The issue, then, is not, what is the best way to teach, but what is mathematics really all about...controversies about high school teaching cannot be resolved without confronting problems about the nature of mathematics. (p. 34)

In summary, the first implications of a conceptual change theory of knowledge are that the curriculum theorist must reexamine his/her own discipline, and that the selection of content must reflect accurately the theories of knowledge embedded in the discipline. For mathematics, this means that one must investigate the development of mathematical concepts and understand how the methods and content evolved inter-

actively as the discipline grew and progressed. Mathematics should not be taught as a set of finished products in which their evolution is considered unimportant or irrelevant. Because consideration of a discipline and its theory of knowledge requires a command of the discipline, such a task must be undertaken as specific to a subject matter.

The second implication of a conceptual change theory for the selection of content is with respect to particular concepts. Using a conceptual change theory one can examine the evolution of a particular concept. It is not imperative that the student retrace the exact route of development of a concept. It is a weak recapitulation plea suggesting that examining the development of the concept reveals a variety of ways of seeing th concept, and thus ought to be helpful to curriculum development. History also provides at least one route by which the concept can be developed in its entirety up to the present.

A quick example in mathematics is the development of the concept of function. This concept is considered significant to modern mathematics, and yet, it is presented to students usually in a definitional form, either as a particular type of relation between sets, or as a graph or rule which requires unique pairing in the range for any element in the domain. Seldom is the evolution of the concept considered. The concept of function, however, developed in response to three interacting demands on mathematics in the 1600's. One, the dominance of geometry began to wane, as algebra began to develop. One difference between algebra and geometry was that algebra focused on the development of methods of solution, whereas geometry traditionally investigated

properties of mathematical objects. The function concept begins its evolution as this juncture. The second influence on the development of the function was the introduction of the variable. Geometry is primarily static; as mathematics strove to deal with change, the variable became of prime importance. Finally, scientific inquiry blossomed in the 1600's so that variability began to include applications such as temperature, growth, vibration, rate, acceleration and so on, and the function concept was able to describe these occurences more accurately. Hence, it is the combination of symbolism, scientific pressure, and a change in the object of inquiry which provoked the development of the concept of function. Little of this is communicated to students, and the function concept becomes simply another rule and label to be memorized. The way we assess whether a student "has the concept" or not turns out to be based on a relativity trivial characteristic of 'function' the uniqueness of the range-domain pairing.

So, the second implication of a conceptual change theory of knowledge in one's discipline allows one to select content that represents the discipline accurately and to examine the evolution of concepts to provide a wealth of alternative ways to aid students in learning the new concept. By being aware of the alternative concepts, one can more easily diagnose difficulties attributable to students concepts and may also choose to introduce students to a variety of these concepts and discuss the strengths and weaknesses of each for particular situations.

Selection of Content in Various Subject-Matters

In review, the overall implications for curriculum theory which is suggested in the discussion of the selection of content is that the

theory of knowledge implicit in a discipline needs to be examined. Careful and critical examination of epistemological assumptions in the discipline provides the parameters for the selection of content. This must be undertaken by people with a command of the subject matter and with a concern for the theory of knowledge. Since education provides a context for this concern for the theory of knowledge, it is an essential part of curriculum theory.

The different disciplines will not have the same epistemologies, although some of the basic questions will surely be shared. In order to be clearer about the differences among various disciplines on theories of knowledge, I will provide some illustrations.

One basic difference is apparent in the comparison of the social sciences and the natural sciences (and mathematics). Whereas, in the natural sciences the hierarchy of theories is primarily synchronically, that is over time, with only a few theories competing in a given period of time, in the social sciences competing theories exist diachronically, at the same time. For that reason, in mathematics the development of concepts is stressed over time, whereas for example, in political science or literary criticism, criticism among concepts at a particular time may take priority.

Other variations among the disciplines include the role and stress given power and authority questions, the explicitness with which value questions are included, the relationship between the empirical data and the theory, and the relative importance of sociological and psychological questions. The vehicle for communication of the current values and theories and knowledge of a discipline may vary, so the

importance of language will also be a prime consideration. Finally, the image and function of the discipline in society will influence the different epistemologies.

What remains constant among the disciplines according to a conceptual change theory is the need to explain both the continuity or stability of the discipline and its variations either over time or at a given time. Toulmin (1972), has put it this way:

> An adequate theory of conceptual change must answer the questions...'on what occasions, and by what processes and procedures, does one basic set of collective concepts--in science or elsewhere--come to displace another?' It must answer these questions in a way that explains in one and the same set of terms, both why our ways of thinking in some fields remain effectively unchanged over long periods, and also why in other fields they sometimes change rapidly and drastically." (p. 122)

For example, in mathematics, one must not only account for the changes in the various concepts, but must also account for the relative stability of the discipline over history. So, another dimension along which the disciplines are likely to differ is in their change processes and the rate of change which is likely in the discipline.

Thus, in the selection of content, the curriculum theorist must first possess or obtain an accurate perception of the discipline. Because I am doubtful that such a perspective cannot be gained without a reasonable command of the discipline, this seems to be a necessary condition for undertaking curricular work. In addition, one must examine the discipline for its epistemological assumptions, and this is also a necessary condition for the effective design of curriculum. The arena of education may provide the motivation for such an endeavor, but its successful completion must be done in a subject-matter specific

manner, although it may be guided by general work on theories of knowledge.

The Development of Ways of Making Content Available

The second task which Huebner (1976) suggested for curriculum theorists is "finding ways to make its content available to young people." (p. 165) He continued on to suggest that this task "is primarily a matter of educational technology. The various sciences now associated with education, such as learning theory, child development, and cognitive psychology are most appropriately seen as technical tools for making content available..." (p. 165) To suggest that the adjustment of the content to make it available to young people is a technical task is to ignore the epistemological relationship between what we teach and how we teach it. In this section, I will explore this relationship and propose broad outlines for it according to a conceptual change theory of knowledge.

One way to demonstrate the influence of epistemology on the relationship between content selection and presentation is to analyze learning theories for their epistemological assumptions. This has been done in part by Shulman (1970), Shulman and Tamir (1973), and Petrie (1976). Instead, I will suggest that conceptual change itself has basic principles of learning with which it is consistent, and I will tentatively present some of these principles.

If one accepts conceptual change, then the understanding of what constitutes knowledge requires an understanding of its development within the discipline. The objective is to understand how knowledge changes, not to search for particular pieces of knowledge which appear to remain

static as truth. This attention given to the development of knowledge is the basis of the tie between knowing and learning. Learning can be seen as 'coming to know', and as such, the conceptual change theory has implications for issues surrounding learning. Primary in the relationship are the concepts of 'justification,' 'comprehension,' and 'rationality.'

Petrie (1976) argues for a logical connection between the development of knowledge in collectives and in individuals when he wrote:

> What I want to suggest here is that there is a very strong logical similarity between the general growth of human knowledge on one hand and the *educational* process conceived as a kind of "growth of knowledge" on the other hand. This similarity is *not*, however, that the growth of human knowledge is a social process, and the educational process is an individual instance of this. Both processes include social *and* individual change." (p. 119)

If the development of knowledge collectively does yield insight into student learning, then the following general principles seem to be consistent with our understanding of the development of knowledge:

1. A first principle might be that knowledge is not objective, and static and external to the people who know. As such teaching should not be seen as the transmission of knowledge as though packaged, but as the process through which teachers and students interact in the construction and understanding of knowledge. This means there is a reliance on the development of knowledge in students.

2. A second principle from conceptual change is that different concepts exist at a time and over time. For the task of teaching content, this means we must expect and attend to individual differences, both among individuals and over time.

3. Thirdly, if individual differences at a time are likely, then there will be a need to start "where a student is at." If 'comprehension' is defined as the student making sense of knowledge given his/her present concepts, then this process will be dependent on the

student's previous knowledge.

4. If knowledge is not static, but changes, and at times, those changes require a reconceptualization of previous concepts, then curriculum theorists must try to anticipate when such occurences are likely. Rationality is most likely to be revealed in this process of reconceptualization, so then it warrants considerable attention. Petrie (1976) referred to such change as 'accomodation' and wrote:

> "The point is that by looking *both* at individuals' cognitive schemes and how they have developed *and* at the development of intellectual disciplines we may be able to see the crucial points of conceptual change, compare them, and see how they justifiably be made congruent." (p. 129)

5. In this process of conceptual change, close attention must be given to the reasons for believing what students believe, both the private 'making sense' of what the learning, which I am calling 'comprehension' and the public acceptability of their reasons, which I am calling 'justification'. These two processes comprise 'understanding' as I see it. Why a student believes something, and what is it that that student believes become an integral part of education. In that way, the processes of comprehension (what the student believes) and justification (why s/he believes it) become focal in education, instead of just processes along the way to producing students with adequate stores of knowledge.

6. Finally, this all suggests a major emphasis in schools on reflection on the development of knowledge and criticism of the various competing theories.

The major point to make here is that the "making content available to students" is not simply a function of the characteristics of the learner, nor is it a technological question. Not only does one's understanding of one's discipline affect profoundly what content is selected for presentation, but it also affects how that content is presented. The impact of one's theory(ies) of knowledge is both on the content and method of presentation and the interaction between these two cannot be ignored.

Hersh (1979) made this point about mathematics, and wrote: "One's conception of what mathematics is affects one's conception of how it should be presented. One's manner of presenting it is an indication of what one believes to be most essential in it." (p. 33) Perhaps some illustrations of possible applications of these principles to mathematics will clarify the point.

To take an illustration from calculus, I will contrast the geometric and arithmetic approaches. Historically, calculus developed through a dialectic sequence in which geometric approaches were contrasted to arithmetic approaches. Continuity, a prime concept of calculus was more easily understood geometrically, as in a continuous line, but tenaciously resisted 'arithmetizing' until the concept of a limit or Cauchy sequences were developed. Calculus looks differently from each perspective; some theorems are easier geometrically, and more difficult algebraically. However, modern mathematics has an ethic which advocates for the arithmetizing of calculus and rejects the geometrical approaches as imprecise and informal. They are classified as intuitive and deemed not appropriate for proofs. In the teaching of calculus, this ethic is strikingly apparent. Teachers use graphs to explain concepts to students but repeatedly admonish them not to rely on them for the proofs in exams. Geometrically simple limit problems must be translated to cumbersome algebraic formulae.

The relevance of the conceptual change theory would be to suggest that these two methods could be explicitly contrasted in calculus to show how different concepts in mathematics are possible. Furthermore, the state of development of various students could be assessed to see

which method of presentation is more appropriate for various students at different stages. Sequencing of different types of presentation could be based on an assessment of this. Finally, students should be encouraged to examine their own concepts to determine their preference, and this preference should be a rational selection, not an unexamined one. If mathematics is presented to students in a single, linear manner, then students will believe that mathematics is linear and absolute. This conception must be challenged both in the content which is presented and in the manner of presentation of that content.

In summary, Huebner suggests that the design of curriculum for presentation to students is a technical task, once the content is selected. Such a claim ignores the fundamental interaction between the content and its method of presentation. These two are related through the theory of knowledge one accepts. If one accepts a conceptual change theory of knowledge, then one is committed to certain basic principles in the presentation of content. These include a focus on the development of knowledge in individuals, not on the transmission of knowledge, as well as a recognition that various concepts are possible among students, and in a student, over time. Also, at times, concepts change at which point, knowledge is not accumulated, but reorganized and these changes may be anticipated by consideration of the fit between subject matter and students cognitive development. Careful subject-matter focused attention must be combined with developmental, cognitive psychology in the task of preparing content for presentation to students.

A Call for an Eclectic

In this paper, I have suggested that curriculum theorists reconsider their two basic tasks, the selection of content and the design of a presentation of that content. In doing so, I think that primary attention must be given to the theory(ies) of knowledge which is embedded in the disciplines in which one is working. Although an overriding theory of knowledge is possible, this theory plays out differently in different disciplines. For this reason, the examination of theories of knowledge must be subject-matter specific. In doing so, I am calling for an eclectic view of curriculum. Schwab (1971) has done so, but the character of this differs.

What needs to be undertaken is the examination of a variety of school subjects for the epistemological roots in their disciplines. These various theories of knowledge need to be contrasted as to their relative emphasis, and the merits of applying a conceptual change or a critical theory needs to be assessed. Differences and commonalities among the disciplinary theories of knowledge could be discussed, and the implications of each for selection of content and presentation of that content could be compared. The objective would not be a merging of the various theories, but a clearer specification of what elements might be common, if any, and what the most significant differences are.

The three basic assertions put forth in this paper are as follows:

1. The examination of the theory of knowledge implicit in a discipline merits consideration for its necessary impact

on curriculum, both in content selection and presentation.

2. A conceptual change theory reveals the integral ties between knowing and learning. An examination of these ties is important to education, and a part of curriculum.
3. The examination of these issues can be undertaken in a variety of disciplines simultaneously and cooperatively, however, the integrity of each discipline must be protected by the pursuit of a delineation of shared concepts and independent concerns.

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